Construction de profils Sinus Balayés sur Bruit pour la qualification de composants moteurs

Obtaining a Swept Sine on Random Vibration profile for powertrain mounted component qualification

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Résumé

Dans le cadre de la conception et de la validation des constituants montés sur groupe motopropulseur, il est essentiel de connaître et de pouvoir spécifier aussi précisément que possible l'environnement mécanique, et en particulier vibratoire, dans lequel ils seront amenés à fonctionner. C'est à cette condition que l'on pourra par la suite optimiser ces constituants vis-à-vis de leur résistance aux sollicitations liées au fonctionnement du moteur d'une part, ainsi qu'à celles issues du profil routier d'autre part, tout en limitant les coûts et les délais de développement.

L'article présente une méthodologie permettant de combiner simultanément un sinus balayé et une densité spectrale de puissance d'accélération pour s'assurer de la tenue mécanique des composants montés soit sur le moteur soit sur la boîte de vitesse. La première partie traite de la séparation des composantes harmoniques et aléatoires par le biais d'une extraction d'ordre sous forme de signaux temporels. La seconde partie présente une méthode permettant d'estimer le spectre de réponses extrêmes et le spectre de dommage par fatigue d'un profil combinant un sinus balayé et une densité spectrale de puissance. Dans la troisième partie, axée sur la démarche de personnalisation des essais, ces deux composantes sont traitées indépendamment l'une de l'autre puis combinées pour définir une sollicitation de type sinus balayé sur bruit intégrant un usage client. Enfin, des applications de cette démarche sur des cas industriels relatifs à des moteurs automobiles sont décrites.

Abstract

As part of the design and validation of engine-mounted components, it is essential to be able to specify precisely the vibratory mechanical environment in which they will operate. The scope is to optimize the reliability of such components submitted to loads coming from both the engine and road profile. It also allows reducing development costs and time scales.

This article presents a methodology which enables to superimpose a swept sine and a power spectral density of acceleration to check the mechanical durability of engine- or gear box-mounted components. The first part covers the extraction of the deterministic parts from the random process using an order tracking method in the time domain to obtain the wave form of the dominant engine orders. The second part presents a mathematical process for the assessment of fatigue damage and extreme response spectra of a Swept-Sine-On-Random profile. In the third part, based on the test tailoring approach, the harmonic and random parts are superimposed to define a Swept-Sine-On-Random loading function that includes the customer's engine usage profile. Finally, some use cases showing the application of this methodology on industrial engines are described.

Glossary

PSD(A): Power Spectral Density (in Acceleration)
ESUP: Engine Speed Usage Profile
FDS: Fatigue Damage Spectrum
ERS: Extreme Response Spectrum
TVDFT: Time Variant Discrete Fourier Transform
DFT: Discrete Fourier Transform
IIR: Infinite Impulse Response
FFT: Fast Fourier Transform
DOF: degree of freedom

Content

Introduction

The aim of this paper is to present a way of deriving a vibration specification representing an automotive engine environment. Such an environment is best represented with a Swept-Sine-On-Random profile. The vibration specification is meant to be representative of the loading and therefore the fatigue damage that the component shall experience during its whole supposed life. Today, such vibration specifications for an automotive engine are often represented as random Power Spectral Densities or Sine Sweeps. The main advantage of using a Swept-Sine-On-Random definition is found in the very nature of the signal generated: a deterministic component coupled with a random process. As a matter of fact, engine induced vibrations are a mixture of deterministic signals due to its architecture and some random vibrations.

The first section of this article will focus on extracting engine harmonics. Different techniques are compared in order to select the most appropriate one to help define a Swept-Sine-On-Random vibration profile. The second section describes how to estimate both the Fatigue Damage and Extreme Response Spectra for a Swept-Sine-On-Random vibration profile. The third section describes a methodology to build a Swept-Sine-On-Random qualification specification for an automotive engine from vibration measurements and a usage profile. Finally, the fourth section gives some examples.

1 Order Tracking

1.1 Engine Orders (or Harmonics) and Order Tracking

Internal combustion engines used in the automotive industry are rotating machines that generate vibrations. These vibrations are due to detonations in cylinders, balancing issues or misalignment of rotating axes. Vibrations vary proportionally with the speed of the main shaft. Multiples of the fundamental speed are called orders or harmonics.
Order tracking is a technique to analyze the non-stationary noise and vibration signals on rotating machinery. The objective is to identify the various orders that are in a signal and to extract their characteristics. Applications are in acoustics, vibrations, or fatigue.
The most popular and easiest way to look at orders is using a waterfall plot, also referred to as a Campbell diagram. Figure 2 shows an example Waterfall plot with orders represented as lines of high energy, with the slope indicating the value of the harmonic.

Another typical result of the order extraction is the order slice plot. It shows amplitude or phase as function of RPM. Often it is interesting to see the most critical orders compared with the overall spectrum (showing the whole vibration energy) as in figure 1 below.

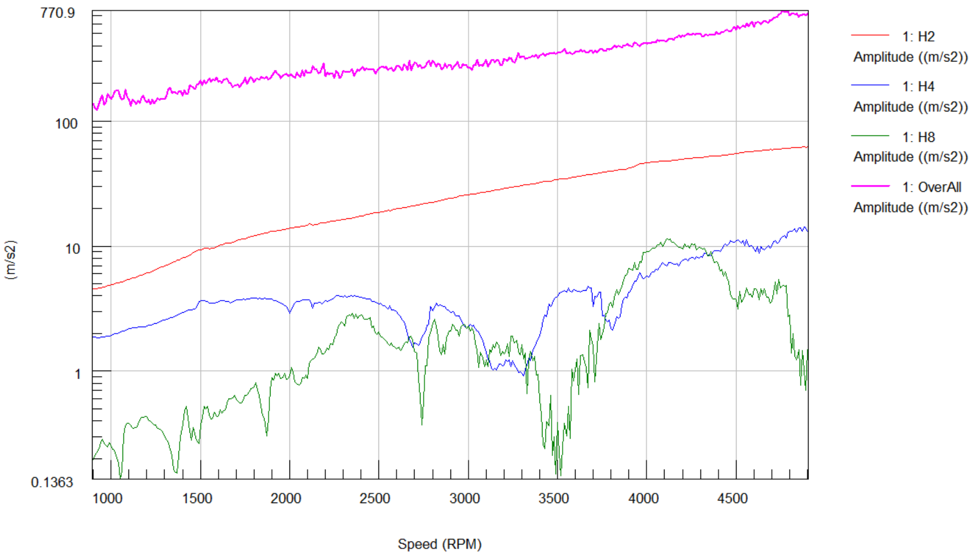


Figure 1: Overall (magenta) with 2nd (red), 4th (blue) and 8th (green) orders

In environmental testing, order tracking filters allows splitting a signal into its harmonic and random parts. This helps when building a Swept-Sine-On-Random test specification. This type of specification is known to be very representative of an automotive engine. Order analysis helps build this specification since the main orders represent the harmonic content defined as swept sines, while the rest of the signal represents the random part of the process. There are different ways of extracting orders. This is the subject of the next section.

1.2 Presentation of usual Order Tracking Techniques

<u>Waterfall of Spectra</u>

The Waterfall of Spectra is the most popular method. It is easy to understand and is widely used. It however suffers from various limitations.
This method performs sliding FFTs on time domain data. The average RPM over which the transform is performed is also calculated. The average RPM is then used to estimate the frequency of the orders of interest for each estimated spectra. The underlying assumption in this type of analysis is that the frequency change within a single time block is small, so that the assumption of stationarity necessary for Fourier transform is not largely violated. If the frequency of a periodic component changes within the time block, the FFT will produce an error called smearing. The energy is distributed rather than concentrated; the peak value is different from the real value at the frequency of interest.
There is indeed a trade-off between frequency and speed resolution, referred to as the uncertainty principle. A larger buffer of data points will give a good frequency resolution, whereas a shorter buffer better accounts for speed changes. Ideally, it would be useful to have a large buffer size for low speeds and a short buffer size for high speeds.
GlyphWorks can obtain the Waterfall Display and let the user extract the corresponding spectra, as illustrated in Figure 2 below.

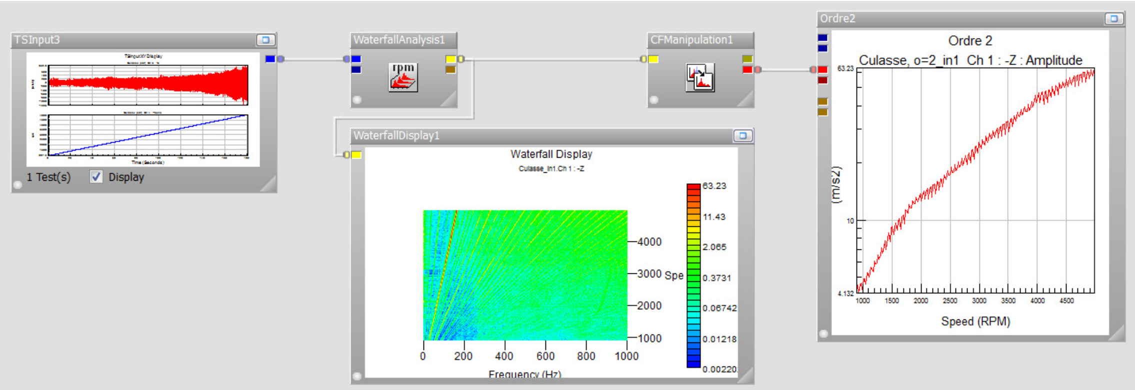


Figure 2: Example Order tracking using spectra waterfalls

The tachometer signal has to be as clean and smooth as possible and have no dropouts. For example, it is suggested in the literature [8] to fit it through a Cubic Spline smoothing algorithm.

This approach is accurate only for low speed and low slew rates. It is however widely used and very efficient for quick trouble-shooting.

Revolution-Based Resampling

This method resamples constant delta-t time series to constant angular intervals. The time at which the equal angular intervals occur are computed by integrating the tachometer signal. The obtained angle domain data is processed through the use of FFTs. The obtained spectrum is therefore showing orders (inverse of revolutions) on the horizontal axis. There is virtually no leakage because if the user decides to resample at 2^n points per revolution, the FFT algorithm will always pick an integer number of revolutions, so we can consider that this data is periodic. Doing an FFT of data sampled in the revolution domain also solves the problem of frequency smearing. Analyzing data in this manner means performing the FFT over a shorter time as the rpm increases: this helps to reduce the errors at high speeds and slew rates. The Revolution-based Resampling technique may prove more computationally intensive but offers leakage-free order slice plots with a very good accuracy. This approach is a constant-order bandwidth order estimation.

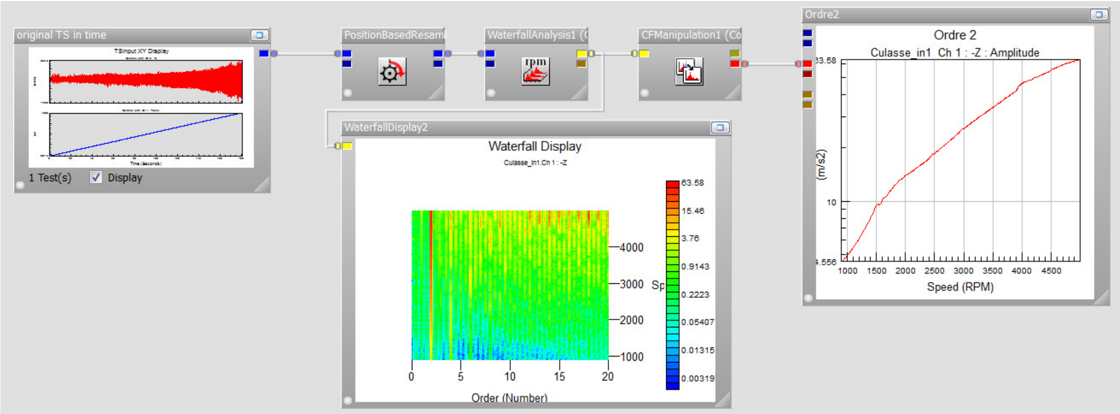


Figure 3: Example order extraction from Revolution-based resampling

It is possible to filter out a specific order by simply band pass filtering the resampled signal around the desired order, and then by resampling it back into time domain.

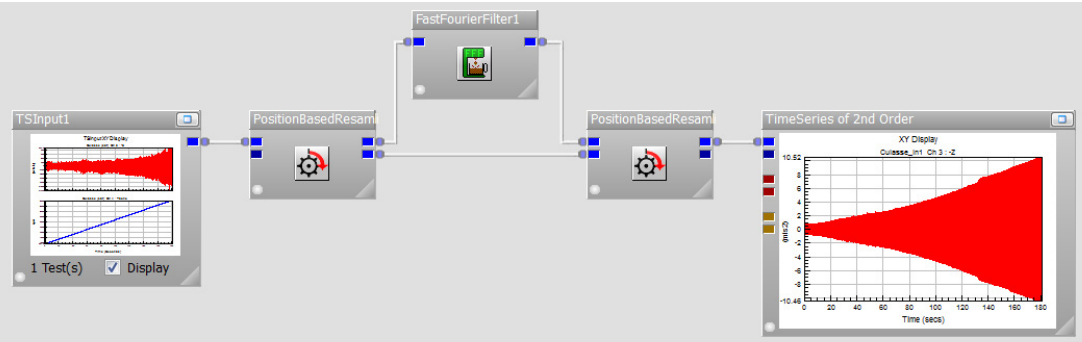


Figure 4: Extraction of the order Wave Form from a Revolution-based resampling technique

Time Variant Discrete Fourier Transform (TVDFT)

This method extracts orders as slice plots directly from the time series. The algorithm is very similar to classical DFT. The TVDFT is defined as a discrete Fourier Transform with a kernel whose frequency varies as a function of the RPM channel. Equations 01 and 02 below compare the TVDFT algorithm with the classical DFT algorithm:

TVDFT

$$a_m = \frac{1}{N}\sum_{n=1}^{N} x(n\cdot\Delta t)\cdot\cos\left(2\pi\int_{0}^{n.\Delta t}\left(o_m\cdot\Delta t\cdot\frac{rpm(n\cdot\Delta t)}{60}\right)dt\right)$$

$$b_m = \frac{1}{N}\sum_{n=1}^{N} x(n\cdot\Delta t)\cdot\sin\left(2\pi\int_{0}^{n.\Delta t}\left(o_m\cdot\Delta t\cdot\frac{rpm(n\cdot\Delta t)}{60}\right)dt\right) \qquad [01]$$

Where N is the total number of data points,
x(t) is the input signal,
Δt is the time between two samples,
rpm is the engine speed,
o_m is the engine order.

DFT

$$a_m = \frac{1}{N}\sum_{n=1}^{N} x(n\cdot\Delta t)\cdot\cos(2\pi\cdot f_m\cdot n\cdot\Delta t)$$

$$b_m = \frac{1}{N}\sum_{n=1}^{N} x(n\cdot\Delta t)\cdot\sin(2\pi\cdot f_m\cdot n\cdot\Delta t) \qquad [02]$$

Where N is the total number of data points,
x(t) is the input signal,
Δt is the time between two samples,
f_m is the frequency.

The parameters are: the order to extract, a bandwidth, a window function, an overlap and a number of engine revolutions for the transform (i.e. an analysis block size).

The transform is made over the number of time points required to achieve the desired order resolution. The order resolution is defined as the inverse of the number of revolutions in the analysis block. This approach is very similar to the revolution-based resampling technique, where more data points will be inserted at low speeds. This transform is considered to estimate an order with a constant order bandwidth.

In GlyphWorks this technique is currently available through the use of a Scripting glyph. An example work-flow is illustrated in Figure 5 below:

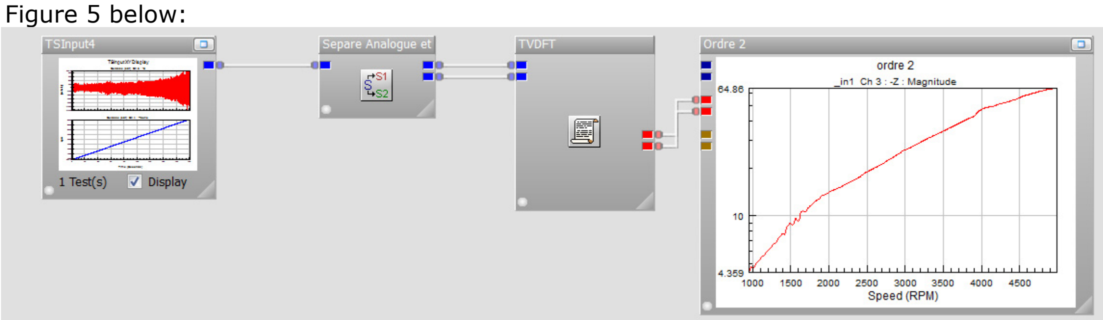


Figure 5: Example Order tracking using TVDFT

The TVDFT has most of the advantages of the resampling-based method, while reducing the computational load considerably.

Adaptive Recursive Filter

The authors have developed an adaptive band-pass recursive filter of selected width, which is centered at a frequency controlled by the RPM channel. The filter's bandwidth can be constant frequency or constant order. The number of poles can control the selectivity of the filter. The output is thus a new signal band-pass filtered around a particular order.

The equation of a recursive filter (IIR) is given in Equation 03:

$$y(n) = \sum_{k=0}^{M} b_k \cdot x(n-k) - \sum_{k=1}^{P} a_k \cdot y(n-k) \qquad [03]$$

Where $x(n)$ is the input signal,
$y(n)$ is the output signal,
a_k and b_k are the Auto-Regressive and Moving-Average coefficients.

Such a filter is called "tracking Filter" because its cut-off frequency f_c is not constant but rather a multiple of the engine speed. a_k and b_k coefficients are therefore dependant on $f_c(t)$. Equation 03 can actually be written as Equation 04 below:

$$y(n) = \sum_{k=0}^{M} b_k(f_c(t)) \cdot x(n-k) - \sum_{k=1}^{P} a_k(f_c(t)) \cdot y(n-k) \qquad [04]$$

Advanced digital signal processing techniques help determine the a_k and b_k coefficients, functions of $f_c(t)$ of the filter's characteristics. Note that the non-linear phase introduced by such an IIR filter can be cancelled by filtering the signal twice, firstly in a forwards direction and then backwards.

This algorithm was implemented within GlyphWorks. User-defined parameters include the order to extract and the bandwidth of the band-pass filter. Such an adaptive filter is able to extract the wave form of an order. The spectrum of the order slice is obtained by calculating the FFT of the extracted time signal as illustrated in Figure 6 below.

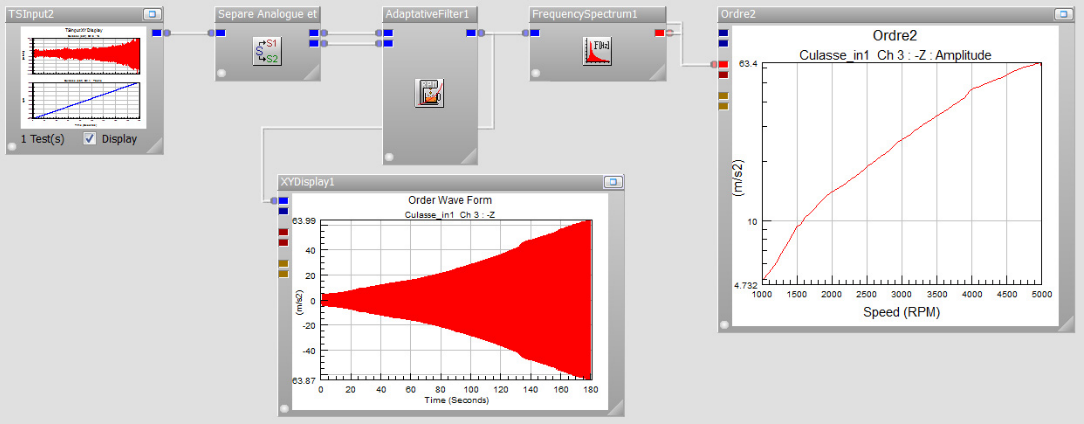


Figure 6: Example Order Tracking using an Adaptive Filter

Other Existing Methods

Other order extraction techniques exist like the Vold-Kalman filtering process. This last approach has many advantages especially when orders are close or crossing each other. This method was not investigated as part of this work because its advantages are not essential for our application. Moreover, it also shows drawbacks like the *a priori* knowledge of the data required for the right parameters to be picked and the possible computational load needed.

1.3 Comparison of Order Tracking Techniques and Conclusion

Criteria

In order to choose what method is best for our needs, it was decided to classify them with simple criteria:

- Accuracy at both low and high speed
- Ability to extract the wave form of an individual order
- Ease of use
- Computer efficiency

Some details about each of these criteria are given hereafter.
Note that the ability to separate crossing orders (i.e. consider multiple uncorrelated input shafts) was not retained as a criterion, as it is not relevant in the present application.

Accuracy
At low engine speed, orders are very close in frequency. Close orders may cause transient beating events. It is therefore important to have a good frequency resolution at low speed in order to correctly separate the orders, which are very close in frequency. At high speed, on the contrary, the slew rate can be very high (gear shift, transmissions) and a short analysis buffer is needed to capture these fast transitions.

Some methods extract orders in a constant frequency bandwidth, whereas others consider a constant order bandwidth.

The difference between the two approaches is illustrated in Figure 7 below. A general recommendation is to prefer a constant order bandwidth.

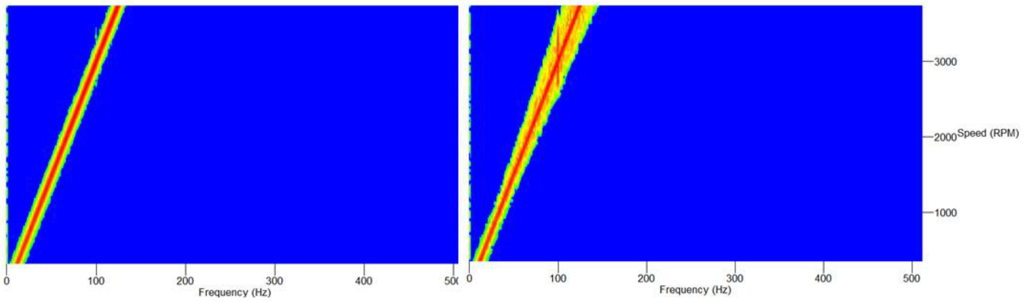


Figure 7: Constant Frequency (left) vs Constant Order Bandwidth (right) strategies

Ability to extract the wave form of an individual order
Time Series data contains the complete information. Time Series can be listened to or subtracted from the total engine signal to evaluate sound quality aspects. It can be Rainflow cycle counted in order to obtain the damage contribution of various orders. Time series allows clear visualization of the non-stationary transitions during a transmission (e.g. gear shift). Some other applications like rotational-induced anomaly correction also require the wave form extraction.

Ease of use
Order Extraction can become quite complicated in some cases. Users are looking for a robust method with few parameters. The parameters must be easy to understand and to set up. Limited a priori knowledge of the phenomenon should be required.

Computer efficiency
It is important for an algorithm to give quick answers so that one can easily modify the parameters or evaluate different scenarios; this is why this criterion was kept.

Results

Below is a table summarizing all advantages (green ticks) and drawbacks (red crosses) of various the techniques mentioned above.

Criterion / Technique	Accuracy	Wave Forms	Easy to Use	Performance
Waterfall of Spectra	✗	✗	✓	✓
Revolution Based Resampling	✓	✓	✓	✗
TVDFT	✓	✗	✓	✗
Adaptive Filter	✓	✓	✓	✓

Conclusion

Generally speaking, it is recommended to start with a classical Waterfall of FFTs for a quick assessment of the phenomenon contained in the signal. Then, when it comes to accurate order extraction, methods such as Revolution Based resampling or TVDFT are powerful as well as very accurate.
A new method is introduced; a simple and robust adaptive filter, which has proven to combine accuracy, ease of use and efficiency. It allows extraction of the time wave form of a specific order, allowing the separation of the dominant harmonic parts and the random part of a signal.

2 Estimate of Fatigue Damage and Extreme Response Spectra for a Swept-Sine-On-Random Vibration Test

Fatigue Damage and Extreme Response Spectra are part of the Test Tailoring method, also known as Accelerated Testing. The global Accelerated Testing methodology is described in references [3], [9] and [17]. The assumption is made that the reader is familiar with FDS, ERS and their use. The following methodology has been validated in the high-cycle domain.

2.1 Estimate of Fatigue Damage Spectrum for a sine tone

The fatigue damage for a simple sinusoidal loading function is given in Equation 05. The damage from a sinusoidal loading function is not calculated the same way as the damage from a narrow-band random process. The narrow-band random process is stochastic whereas the sinusoidal loads are strictly deterministic. The narrow-band random considers a narrow range of frequencies whereas the sinusoidal loading is at a singularly defined frequency.

$$D = \frac{f \cdot T}{C} \cdot S_A^b \qquad [05]$$

Where f is the frequency of the sinusoidal load of amplitude S_A and duration T seconds. The terms C and b are the coefficient and exponent of the Basquin fatigue curve respectively as illustrated in Figure 8

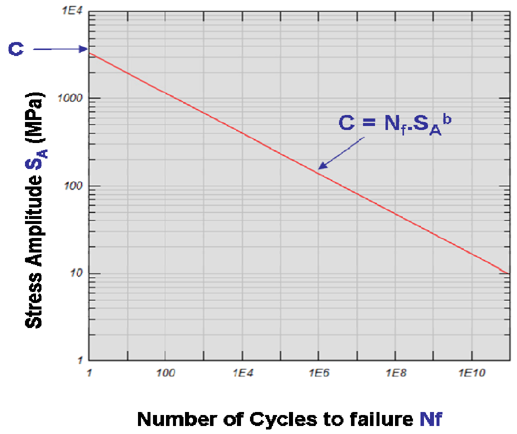


Figure 8: Typical SN Fatigue Curve

We can estimate the RMS amplitude of an equivalent random signal with comparable damage content by equating the fatigue damage given in Equation 05 with Equation 08 and substituting np = f in Equation 08. The equivalent narrow-band random signal is therefore given as Equation 06.

$$rms_{eq} = \frac{\sigma_A \cdot 2^{1/b}}{\sqrt{2}} \cdot \Gamma\left(1 + \frac{b}{2}\right)^{-1/b} \qquad [06]$$

Where rms_{eq} is the equivalent narrow-band random stress amplitude.

The moments of area of the equivalent random signal are determined from Equation 07.

$$\left(m_{eq}\right)_n = rms_{eq}^2 \cdot f^n \qquad [07]$$

Where $(m_{eq})_n$ is the n^{th} statistical moment equivalent to the random process, f is the sine frequency.

Recall for the case of a Narrow-Band process:

From Lalanne [3], an estimate of Damage for a Narrow-Band process is given by Equation 08.

$$D = \frac{n_0 \cdot T}{2C} \left(\sqrt{2} \cdot rms\right)^b \cdot \Gamma\left(1 + \frac{b}{2}\right) \qquad [08]$$

Where $\Gamma(z)$ is the Gamma function defined as $\Gamma(z) = \int_0^\infty t^{z-1} \cdot e^{-t} dt$.

2.2 Estimate of Fatigue Damage Spectrum for a Sine-On-Random

The fatigue damage assessment for sine-on-random loading is made by summing the moments of area of the random PSD with the moments from the sine tones. The combined moments are then substituted in Equation 9 and the fatigue damage assessed in exactly the same way as for the general random case.

Recall: properties of statistical moments:

According to Rice [12], values for RMS, irregularity factor (r), expected number of 0-crossings (no) and expected number of stress cycles occurring in 1 second of exposure (np) can be estimated from moments of area of the PSD. The properties are given in Equation 09.

$$rms = \sqrt{m_0} \qquad n_0 = \sqrt{\frac{m_2}{m_0}}$$
$$n_p = \sqrt{\frac{m_4}{m_2}} \qquad r = \frac{m_2}{\sqrt{m_0 \cdot m_4}} \qquad [09]$$

Where m_n is the n^{th} moment of area of the PSD about the zero frequency axis defined as: $m_n = \int f^n \cdot G_s(f) df$.

$G_s(f)$ is the value of the single-sided PSD of stress at frequency f.

2.3 Calculation of FDS for Sine-On-Random loading

The FDS of the sine-on-random loading treats the random and sinusoidal components separately. The random component is calculated in the usual fashion by filtering the PSD of acceleration by a single degree of freedom (SDOF) transfer function centered about a resonant frequency f_0 Hz. This is described by Lalanne [4] and expressed in Equation 10. The moments of area are determined for each filtered PSD over the specified FDS frequency range.

$$G_s(f, f_0) = \frac{G(f) \cdot K^{2b}}{\left(4\pi^2 \cdot f_o^2\right)^2 \left\{\left[1 - \left(\frac{f}{f_0}\right)^2\right]^2 + \frac{1}{Q^2}\left(\frac{f}{f_0}\right)^2\right\}} \qquad [10]$$

Where $G_S(f,f_o)$ is the single-sided PSD of stress with respect to frequency f and FDS filter frequency fo.
G(f) is the PSD of acceleration,
k is the spring stiffness coefficient,
Q is the dynamic amplification factor of the SDOF transfer function,
b is the Basquin exponent.

The sinusoidal tones are treated in a similar fashion. The equivalent narrow-band random rms amplitude is filtered by the SDOF transfer function as expressed in Equation 11 and the moments determined using Equation 07.

$$rms_{eq}(f_0) = \frac{S_A \cdot K^b \cdot 2^{1/b}}{\sqrt{2} \cdot (2\pi \cdot f_0)^2} \cdot \frac{\Gamma\left(1 + \frac{b}{2}\right)^{-1/b}}{\sqrt{\left[1 - \left(\frac{f}{f_0}\right)^2\right]^2 + \frac{1}{Q^2}\left(\frac{f}{f_0}\right)^2}} \qquad [11]$$

Where S_A is the acceleration amplitude of the sinusoidal tone of frequency f.
fo is the FDS filter frequency,
k is the spring stiffness,
Q is the dynamic amplification factor of the SDOF transfer function,
b is the Basquin exponent.

The moments resulting from each sinusoidal tone are summed with the random and the resultant fatigue damage is calculated in the normal fashion treating the entire process as a random PSD.

2.4 Calculation of FDS for Swept-Sine-On-Random loading

The Fatigue Damage Spectrum for swept-sine-on-random loading is obtained by dividing the test into a number of discrete sine-on-random tests and then summing the FDS of these tests to create the final swept-sine-on-random case. The duration associated with each Sine-On-Random segment depends on the sweep type chosen (linear, logarithmic …) and the total test duration.

2.5 Calculation of ERS for Sine-On-Random loading

The Extreme Response Spectrum of a sine-on-random can be obtained by summing the ERS of the sine functions with the ERS of the random process.

2.6 Calculation of ERS for Swept-Sine-On-Random loading

The Extreme Response Spectrum for swept-sine-on-random loading is obtained by enveloping the ERS obtained for each sine and summing it to the ERS of the random loading.

3 *Building the Swept-Sine-On-Random Profile*

In a combustion engine, the main mechanical load comes from a few engine orders. These orders are due to primary excitations (piston slap, valve operation, etc.). Dominant orders depend on the engine definition, i.e. number of cylinders, their relative position and the use or not of a balance shaft. It is therefore interesting to evaluate the contribution of these orders to the total damage when creating the test specification, while still considering the other vibratory phenomena. That's why it is proposed to create a specific test specification that overlays Swept Sines on the top of a Random noise. Swept Sines represent the engine dominant orders. Noise contains all other vibration components arising from the engine's operation.

The building of a Swept-Sine-On-Random vibration profile can be described in 3 steps:
- The First step is the most delicate. It consists of precisely extracting the dominant order(s) from the measurement signal. Once the order(s) slice(s) are extracted, an equivalent swept sine that integrates the engine usage profile can be constructed. To do so, we can use either the FDS/ERS approach or a specific algorithm presented in section 3.1.
- In the Second step, the 'leftover' signal is considered. It is made of the original measured signal without the dominant orders. This step consists of using the classical Test Tailoring techniques, based on FDS and ERS to process the 'leftover' signal. Either the spectral or the deterministic approach can be used, depending on the nature of the signal. Eventually, an equivalent PSD is obtained, which again includes the engine usage profile. This is covered in section 3.2.
- The last step combines the results of both first steps and includes the test duration reduction. Validating the test duration reduction implies calculating the FDS and ERS of a Swept-Sine-On-Random profile. The methodology allowing calculating the FDS and ERS of a Swept-Sine-On-Random profile is presented in section 3.3.

Note that steps 1 and 2 could be condensed to a single step so that the entire swept-sine-on-random profile is considered, without treating separately the harmonic and the random parts. This would be done by using the algorithms described in sections 2.4 and 2.6. The only difference being that the associated duration from each sine-on-random would come from the ESUP. This approach was not performed here because we wanted to introduce the algorithm in section 3.1 that synthesize the harmonic components into one Swept Sine.

3.1 Processing of the dominant orders

In the first section of this paper, we introduced several order tracking techniques along with methods for extracting order slices in the form of time domain wave forms. For this section, we will only consider the order slice spectra.

Here, the proposed approach has the goal of finding a sine sweep with equivalent damage to the extracted orders, with a given engine usage. For the activity of engine-mounted components, the engine speed usage profile is represented as a histogram showing duration versus engine speed. The engine speed usage profile is a synthesis of the mission profile (or expected usage) coming from a wide number of customers. It incorporates the variability of usage. When looking for the equivalent Sine Sweep, an important parameter is the sweep type: linear sweep (same duration at each frequency) or logarithmic sweep (same duration in every octave). Another important input is what Fatigue law to consider. The Basquin law it is generally accepted. This law makes the implicit assumption that the failure occurs in the high cycle fatigue domain, which is satisfactory for engine mounted components.

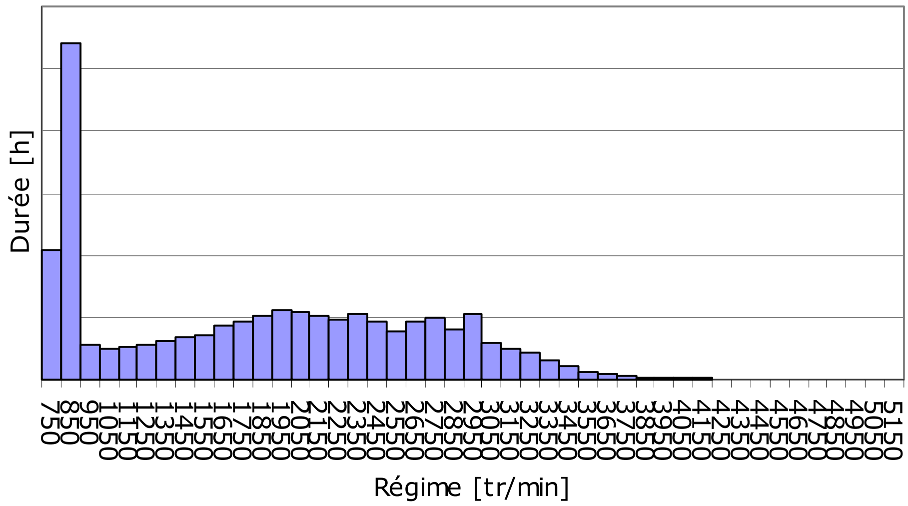


Figure 9: Example engine usage

The methodology described here allows synthesizing several harmonics into one single swept sine. This makes the Swept-Sine-On-Random definition simpler. The methodology used to evaluate a single swept sine, representative of the total usage duration can be broken down in three steps:

- First Step is to calculate for each order and each frequency (or engine RPM, since the order value is known), the duration spent for an iso-damaging acceleration level of 1 m/s², knowing that the engine speed usage profile must be associated. We therefore use the following Equation:

$$T_{ref} = T_{ESUP} \times \left(\frac{\ddot{x}_{measure}}{\ddot{x}_{ref}} \right)^b \quad [13]$$

Where b is the Basquin exponent,
T_{ESUP} is the duration from the engine speed usage profile,
T_{ref} is the calculated duration for $\ddot{x}_{ref}$ fixed at 1 m/s²,
$\ddot{x}_{measure}$ is the magnitude of the extracted order.

This can be rewritten as :

$$T_H(i,f) = T_{ESUP}(f)\left(\frac{\ddot{x}_{measure}(i,f)}{1} \right)^b \quad [14]$$

Where $T_H(i,f)$ is the duration for an acceleration of 1 m/s² that has the same damage as $\ddot{x}_{measure}(i,f)$ for the duration $T_{ESUP}(f)$,
i represents the i^{th} dominant order,
f is the frequency.

- The second step consists of summing the obtained durations for each frequency of each dominant order. It is also necessary to calculate the test duration spent at each frequency. This is done using the following steps:

- Summation of the durations associated to a 1 m/s² acceleration level of each order H(i):

$$T(f) = \sum_i T_H(i,f) \quad [15]$$

- Duration d per frequency f in the case of a linear sweep between f_0 and f_1 :

$$d(f,T) = \frac{T}{f_1 - f_0} \quad [16]$$

Where T is the total test duration.

- Duration d per frequency f in the case of a logarithmic sweep between f_0 and f_1 :

$$d(f,T) = \frac{T}{f \times \ln\left(\frac{f_1}{f_0} \right)} \quad [17]$$

Where T is the total test duration.

- The last step uses still Equation 13 to find the magnitude of the equivalent sine sweep for a duration d from the total duration T (obtained in equation 15). The following Equation is used:

$$\ddot{x}(f) = 1 \times \left(\frac{T(f)}{d} \right)^{1/b} \quad [18]$$

Where $\ddot{x}(f)$ is the equivalent sine sweep.

This methodology derives an equivalent swept sine without calculating any Fatigue Damage Spectrum or Extreme Response Spectrum.

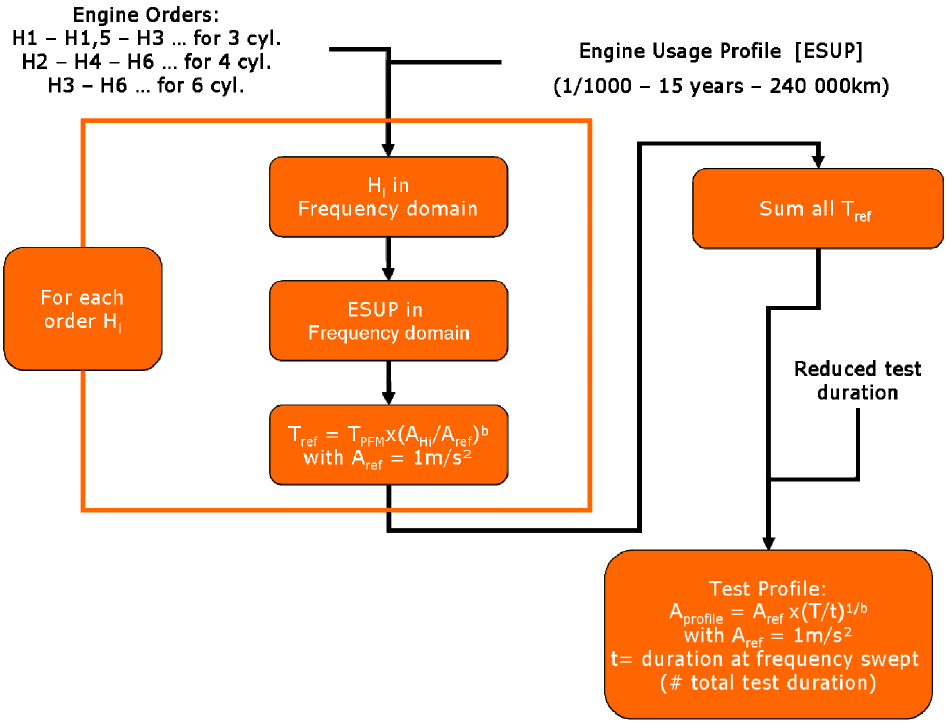


Figure 10: Synoptic chart of the methodology to estimate an equivalent Swept Sine

3.2 Processing of the time domain signal without its dominant harmonics

From the order analysis, we can subtract the wave forms of the main orders from the measured signal. The obtained signal contains every other phenomenon that we consider secondary for our application; we will call it the leftover signal. It is however clear that the leftover signal still contains minor orders due to the engine dynamics, which are deterministic by nature. It is therefore not strictly speaking random. Taking its PSD would introduce a strong bias in the results. This is why it was decided to apply the Test Tailoring methodology using a full deterministic approach. Note that we may separate the random and harmonic contributions and treat them separately because the magnitudes of the extracted orders are definitely dominant.

The Test Tailoring methodology used to obtain an equivalent PSD profile requires the analysis of vibration data coming from a slow engine run-up starting from roughly 1000 RPM up to the maximum engine speed at a given torque. This run-up is cut in time slices of say 100 RPM each. Each slice represents a life situation and the engine usage profile gives the duration a customer would spend in this speed range. This customer's usage should contain a high level of severity. Note that the run-up could be completed with some time at idle, that could be treated as an additional sine-on-random. Each life situation is turned into a Fatigue Damage Spectrum and an Extreme Response Spectrum. A weighting factor is applied to each situation, depending on the duration from the ESUP and the measured signal length for that speed range. A global FDS is obtained by summing all weighted FDSs. This global FDS is then converted to an equivalent PSD to the whole mission profile duration. Note that the use of an equivalent PSD is only strictly valid if the process is known to be from an 'ergodic stationary Gaussian random process'.

The next figure helps the understanding the global methodology for the Test Tailoring of an engine mounted component.

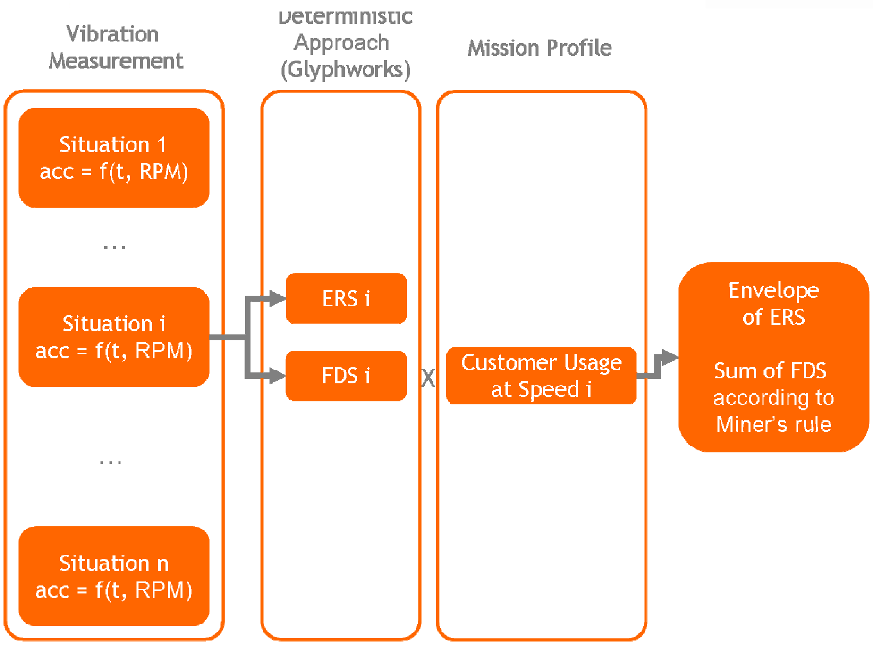


Figure 11: Synoptic chart showing the process of calculating the global ERS and FDS

3.3 Synthesis of a Swept-Sine-On-Random specification

The synthesis allows obtaining a swept-sine-on-random specification for a given reduced test duration. It also allows checking the test duration is not over-accelerated, which would show unrealistic vibration levels. Checking the reduced test duration is made via the 'exaggeration factor'. To obtain this factor, FDS and ERS of the swept-sine-on-random need to be calculated for both the original test duration (the original mission duration) and the reduced one. Spectra obtained for the original test duration will be called Reference FDS and ERS. The methodology used to estimate these spectra is described in the previous section. The exaggeration factor is defined as the rate between the Reference ERS and the ERS of the accelerated test. Generally speaking, a reasonable test reduction involves an exaggeration factor of 2 or less on the frequency range of interest.

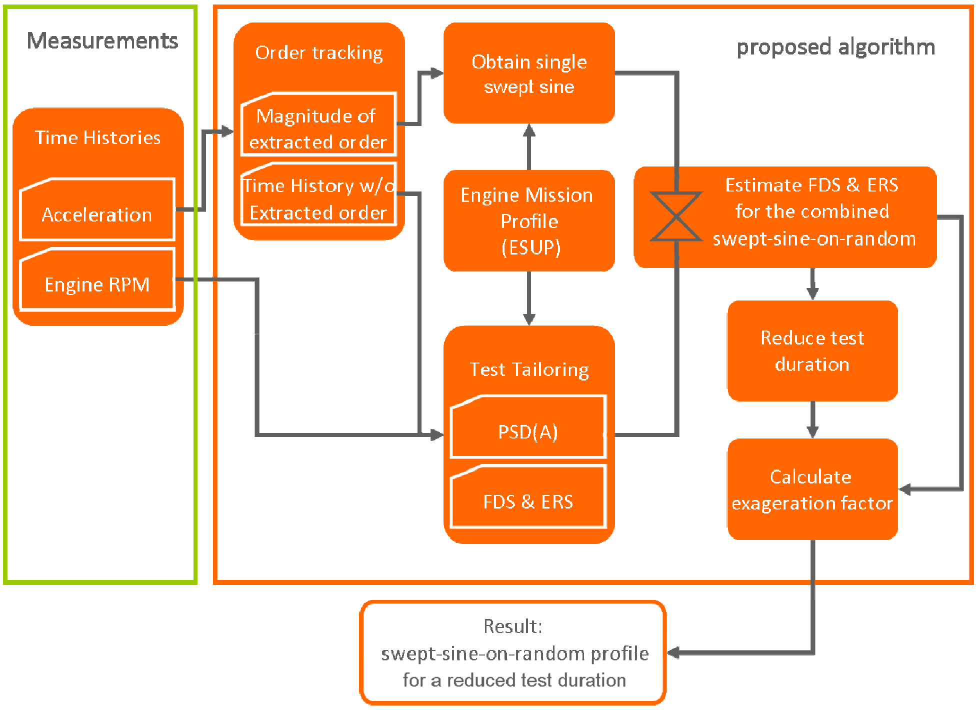


Figure 12 : Synoptic chart of the proposed methodology to get a Swept-Sine-On-Random qualification specification

4 Example Use Cases

We tested the described methodology on several engine vibration environments: a 4 cylinder diesel engine and a 3 cylinder petrol engine. We deliberately limited our scope to only urban usage of these engines, in order to stay on a similar type of usages.

Customer's Mission Profile

The customer's profiles presented here were established for a city usage. Durations are normalized.

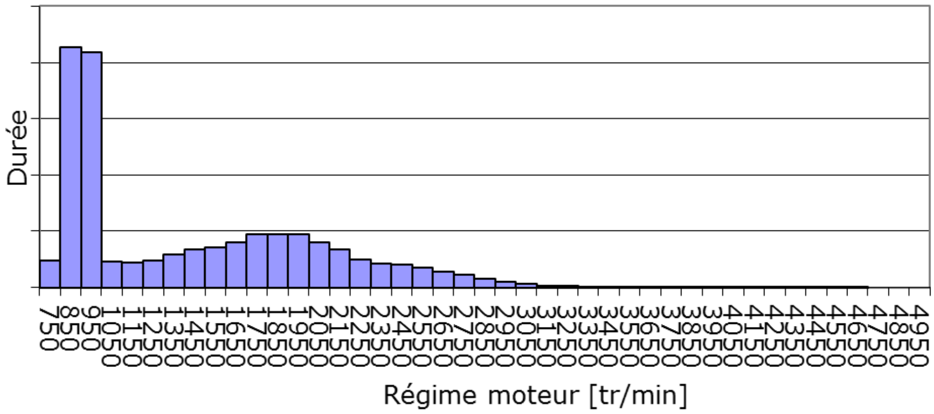


Figure 13 : City usage of a 4 cylinder diesel engine

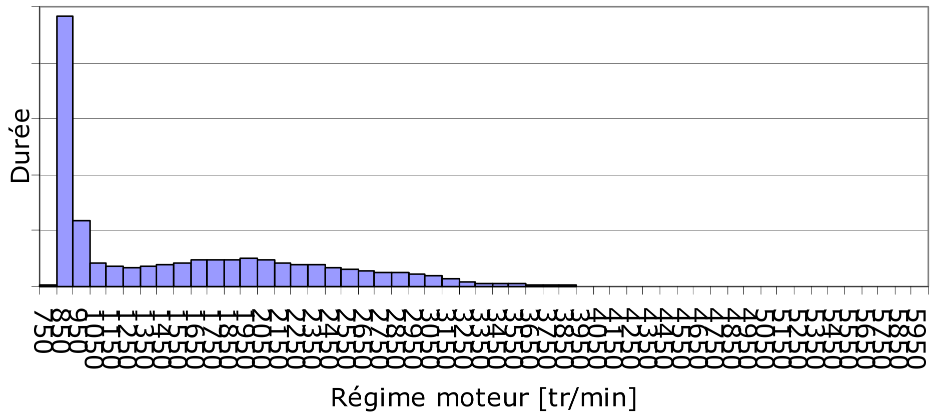
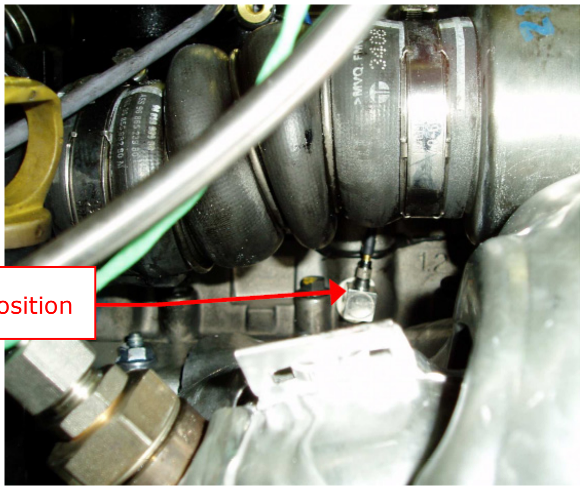


Figure 14 : City usage of a 3 cylinder petrol engine

4.1 Case of a 4-cylinders Diesel Engine

Only the vertical axis of the engine is considered here (pistons' axes). The vibrations were measured on the cylinder head during engine run-up with engine at full load. Picture 01 below shows the engine environment, where the accelerometer was positioned.



Picture 01 : Sensor position on the 4 cylinders Diesel engine

On a 4 cylinder diesel Engine without balance shaft, order 2 dominates. However, for this example we also considered the 4th order.

The first step is the extraction of the 2nd and 4th orders wave forms and order slice spectra. Results from both the adaptive filter and the TVDFT are shown in Figure 16 below. This allows a comparison of the results using 2 different order extraction techniques: both methods give extremely similar order slice spectra.

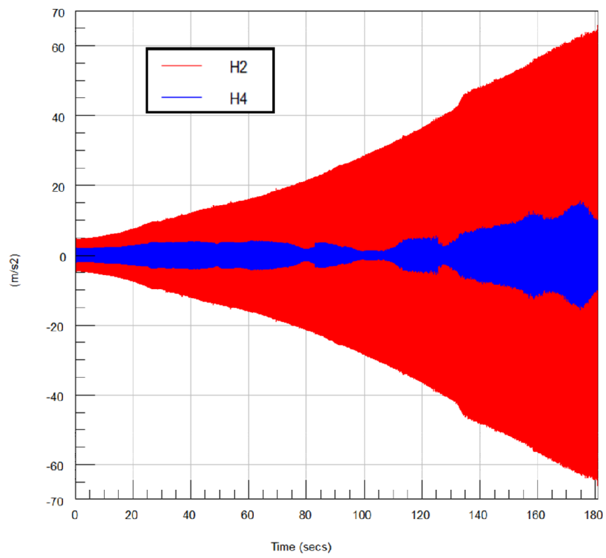


Figure 15 : Wave forms of orders 2 and 4 (adaptive filter only)

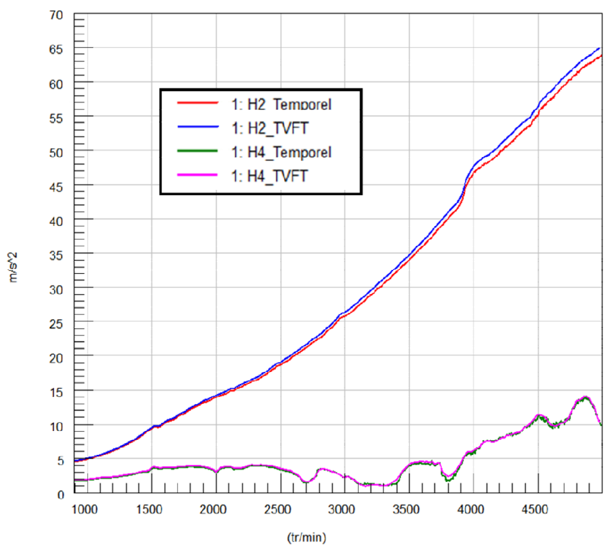


Figure 16 : Orders 2 and 4 slice spectra

From the magnitudes of orders 2 and 4 slice spectra; one can derive a single swept sine using the methodology described in section 3.1. Using the leftover signal (i.e. without the order's contributions), one can derive a power spectral density of acceleration, as described in the traditional test tailoring approach describer in section 3.2.

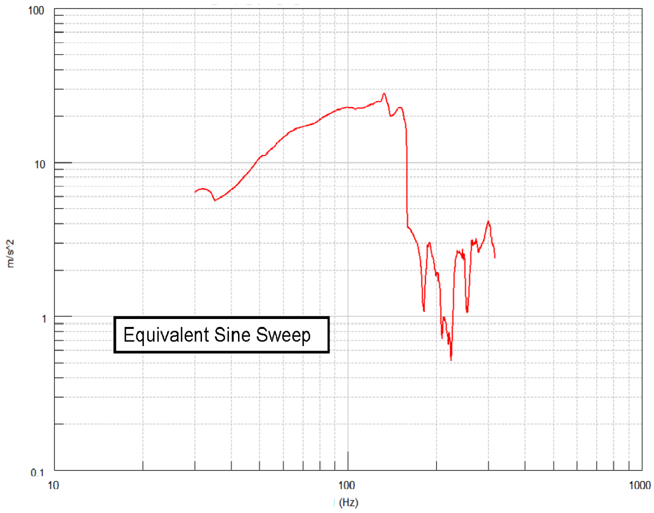


Figure 17 : Swept Sine equivalent to orders 2 and 4

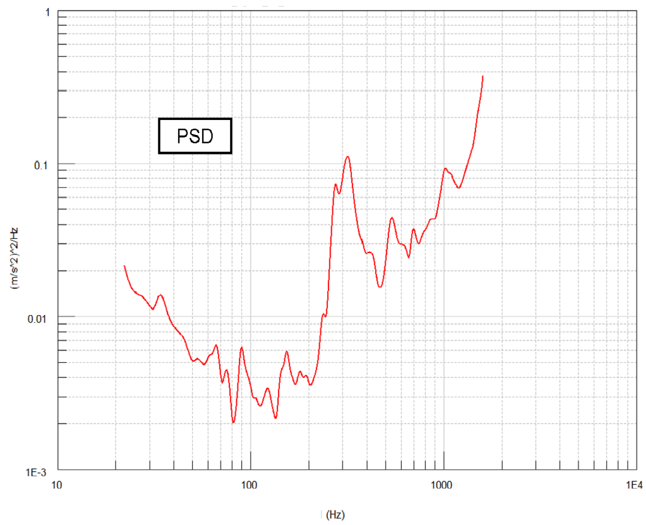


Figure 18 : PSD(A) obtained for the leftover signal

The test reduction is applied on both the swept sines and the PSD. However, Reference FDS and ERS must be calculated first. For the sake of comparison, the FDS of the total time domain signal was also calculated using a full deterministic approach (i.e. 1 DOF filtering, rainflow cycle count, Miner's rule for damage summation). Differences between the Swept-Sine-On-Random FDS and deterministic-based FDS are very little. Both resulting FDS are presented below:

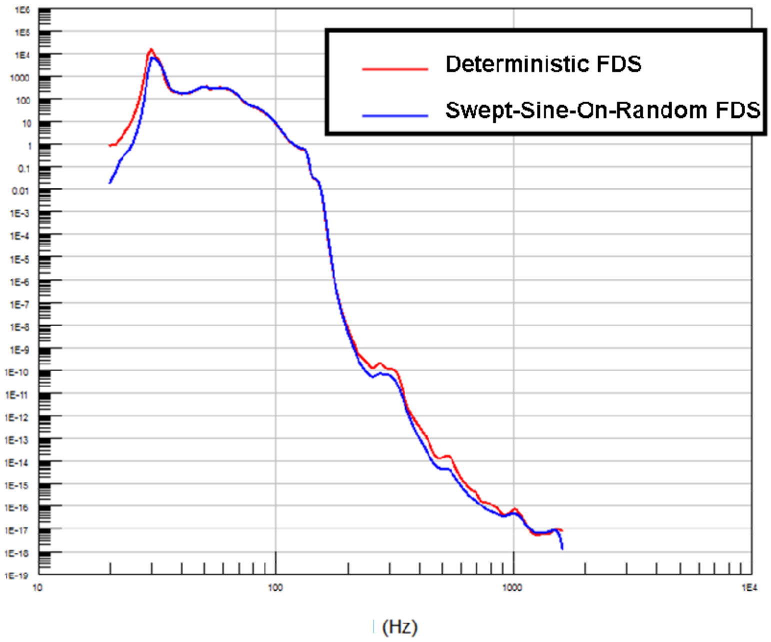


Figure 19 : Comparison of deterministic vs spectral SDF

Once both the swept sine and the PSD are scaled for the reduced test duration, it is necessary to estimate the so called 'test' ERS in order to obtain the exaggeration factor. In the present case, the mission total duration is 7500 hours and the reduced test duration is roughly 100 hours. Figures 20 and 21 below show both ERS obtained before and after test reduction and also the exaggeration factor.

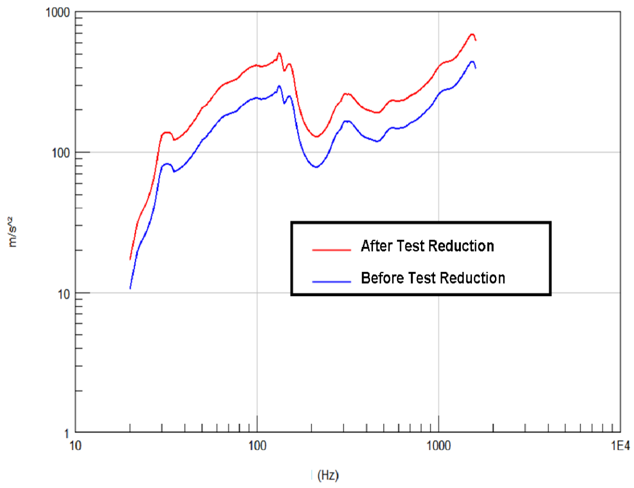


Figure 20 : Comparison between reference and Tests ERS

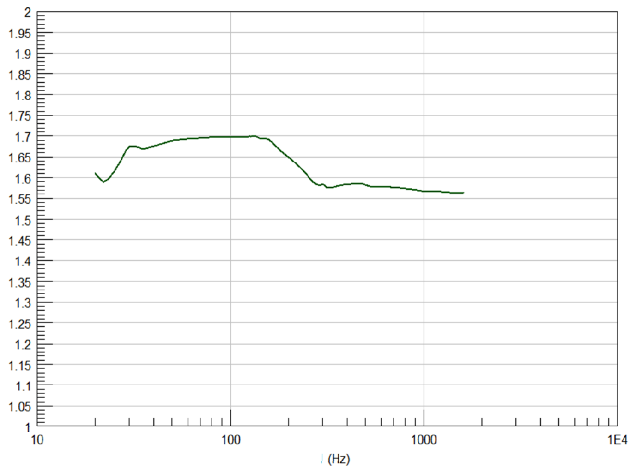


Figure 21 : Exaggeration Factor

The accelerated Sine-On-Random qualification profile is actually a Swept-Sine profile reproduced simultaneously with a PSD profile on the test rig. Figures 22 and 23 below show both profiles.

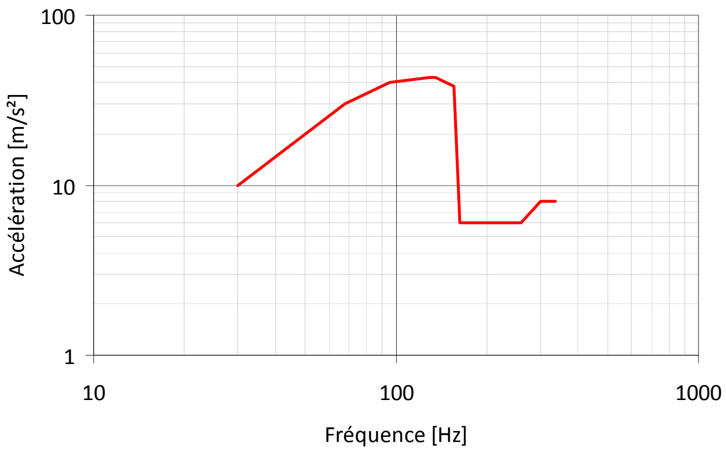


Figure 22 : Swept Sine Profile

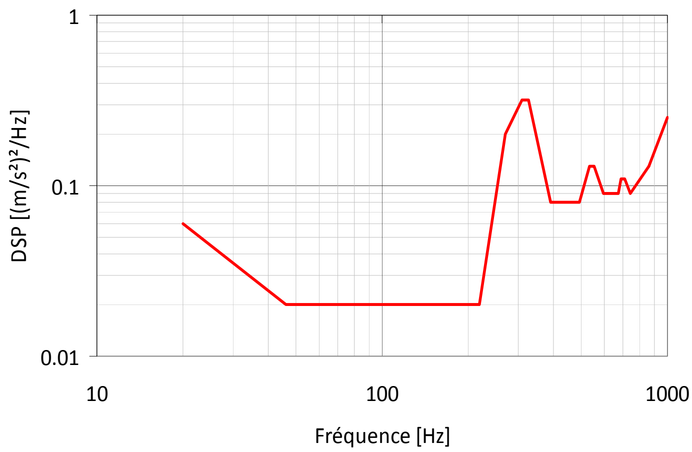


Figure 23 : PSD(A) Profile

Note that, although feasible, the Swept-Sine-On-Random profiles presented in this paper do not include any guarantee coefficient nor test factor for the purpose of comparison. It is however strongly recommended to use them when applying the test tailoring approach.

Also, for the sake of this example, results of a single axis (Z) were presented but the same methodology could have been used for the X and Y axes.

4.2 Case of a 3-cylinder Petrol Engine

Just like in the previous example, only one axis is used here. The data was measured on the cylinder head, in the engine's transverse direction (perpendicular to the crankshaft). The vibrations were measured during engine run-up with engine at full load.

Dominant orders considered are order 1.5 and 3. Their magnitudes are shown below:

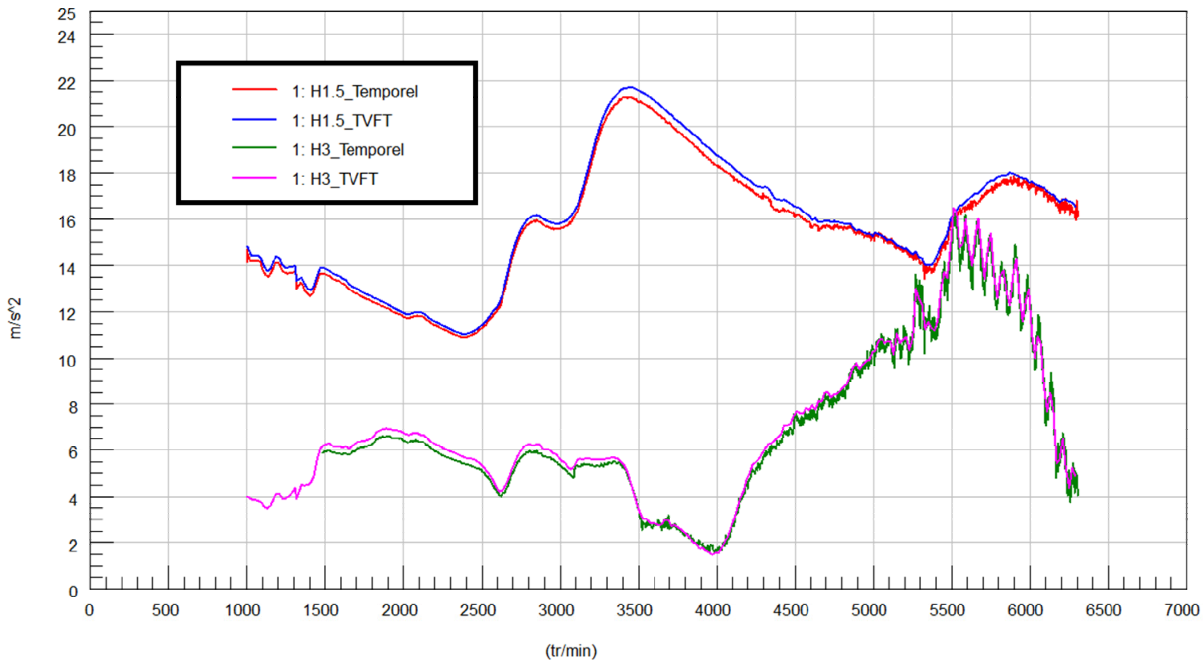


Figure 24 : Comparison of orders 1.5 & 3 obtained using 2 separate tracking techniques

Figures 25 and 26 below show the orders 1 and 3 equivalent Swept Sine and the PSD(A) obtained from applying the Test Tailoring technique on the leftover signal (i.e. without the order's contributions).

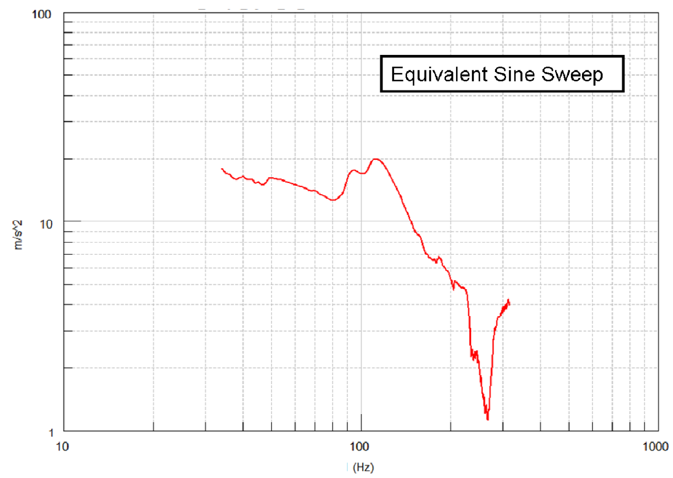


Figure 25 : Swept Sine equivalent to orders 2 and 4

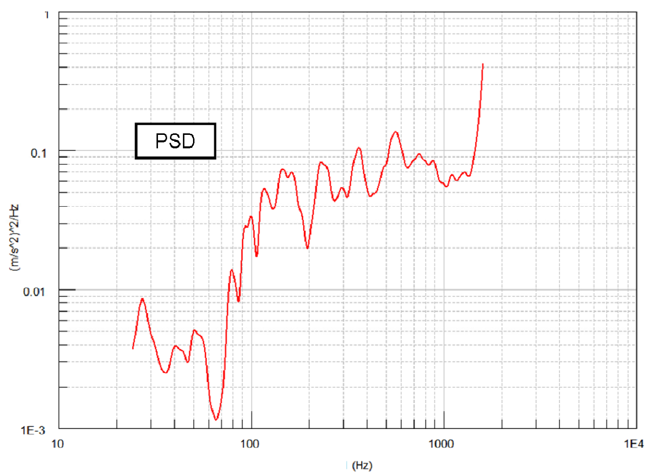


Figure 26 : PSD obtained from leftover signal

Figures 27 and 28 below show the Reference vs. Test ERS and the Exaggeration Factor.

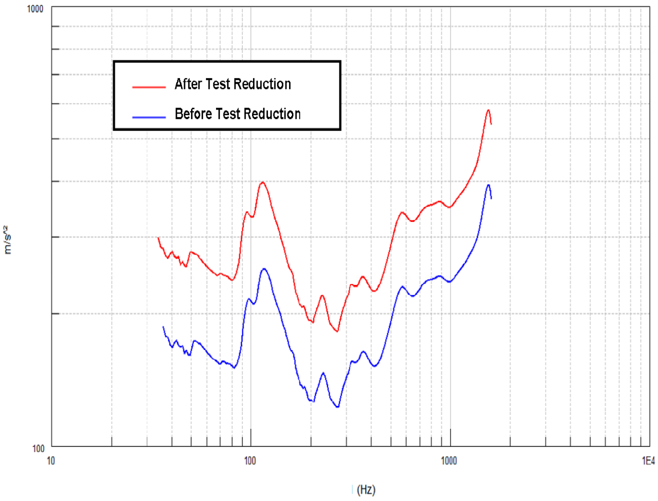


Figure 27 : Comparison between reference and Tests ERS

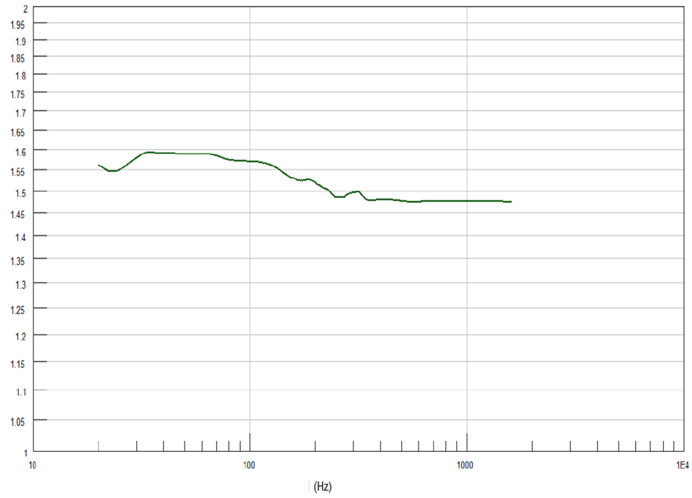


Figure 28 : Exaggeration Factor

The obtained Profiles are shown in Figures 29 and 30 below.

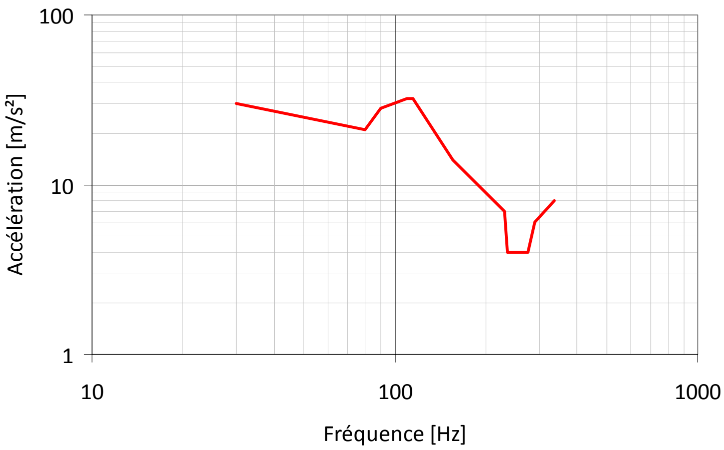


Figure 29 : Swept Sine Profile

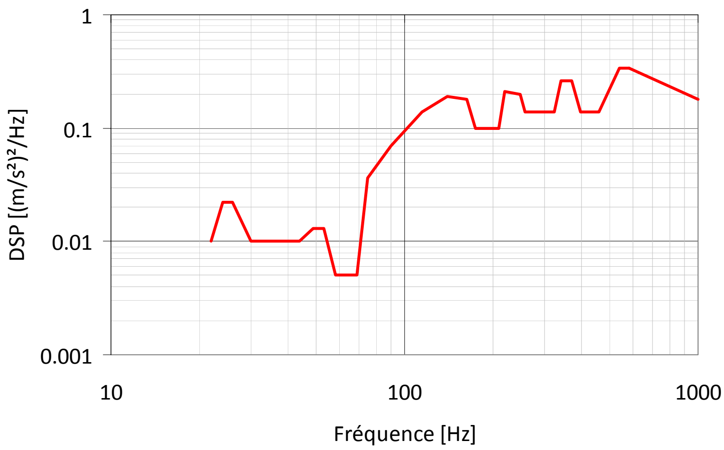


Figure 30 : PSD(A) Profile

Conclusion

The described methodology derives a representative test specification for an engine mounted component by taking into account typical engine vibration data using run-up measurements along with an engine usage profile. The obtained vibration specification is defined as swept-sine-on-random, and incorporates engine usage information.

In order to do so, an adaptive recursive filter was used to track orders in the time domain. The algorithms needed for the estimation of FDS and ERS of a sine-on-random and then swept-sine-on-random environments were presented and applied.

The existing Test Tailoring approach based on Fatigue Damage and Extreme Response Spectra was used here from a new viewpoint, widening its application fields.

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